The Credibility Problem Revisited: Thirty Years on from Kydland and Prescott.*

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Abstract

Macroeconomics research has changed profoundly since the Kydland-Prescott seminal paper. In order to address the Lucas Critique, modelling now is based on micro-foundations treating agents as rational utility optimizers. Bayesian estimation has produced models which are more data consistent than those based simply on calibration. With micro-foundations and new linear-quadratic techniques, normative policy based on welfare analysis is now possible. In the open economy, policy involves a 'game' with policymakers and private institutions or private individuals as players. This paper attempts to reassess the Kydland-Prescott contribution in the light of these developments.

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In memory of Anita Ghatak–a dear friend and a fine technical economist. I think she would have appreciated the keynote lecture and this paper.

1 Introduction

Thirty years ago Kydland and Prescott (1977) introduced the 'time-inconsistency' problem to the economics profession. In a nut-shell, the problem they posed for policymakers is that in a world of forward-looking rational agents, an optimal policy announced at time $t = 0$ ceases to be optimal at every future point in time, $t > 0$. This creates an incentive to re-optimize and renege on earlier policy commitments. The original commitment therefore ceases to be credible. This feature holds even in the absence of uncertainty and even if policymakers are completely benevolent. In other words, if policymakers succumb to the temptation to renege, in a rational expectations world it will be anticipated and, at the same time, can be completely in the interests of the public for which the commitment was made.

The first thing to stress about time inconsistency is that it is a *generic problem* for policy-makers in all areas. For example, for regulated utility services like telecoms there is a classic time inconsistency problem referred to as the 'hold-up problem'. These services require large volumes of investment which, once installed become 'sunk assets' in the sense that most or all of them cannot be removed and used elsewhere or sold on second-hand markets at their original cost. In consequence, private investors are at risk of opportunistic behaviour by regulators particularly over prices, once the investments have been installed; and awareness by private investors of this regulatory risk drives up the required rate of return and the cost of capital. The latter dramatically reduces investment as has been seen in many countries.[1]

However the time-inconsistency problem is mostly associated with macroeconomic policy, and in particular, monetary policy. Following Barro and Gordon (1983) that built on the ideas of Kyland and Prescott, a huge academic literature has grown that has been very influential with policymakers. The central message underlying these contributions are the existence of significant macroeconomic gains, in some sense, from 'enhancing credibility' through formal commitment to a policy rule or through institutional arrangements for central banks such as independence, transparency, and forward-looking inflation targets, that achieve the same outcome.

In addressing these policy issues, until quite recently macroeconomics suffered from two deficiencies: Keynesian models that featured real-world nominal rigidities, although capable of accounting for stylized facts, lacked micro-foundations and were therefore vulnerable to the Lucas Critique. Non-Keynesian models such as the Lucas-supply curve, which lie at the heart of the literature spawned by Barro-Gordon, can be rigorously justified, but fail

[1] See Levine *et al.* (2005)

empirically. The 'New Keynesian Macroeconomics' based on dynamic stochastic general equilibrium (DSGE) models can now claim to reconcile rigor and empirics.

In the essentially static model used in much of the earlier literature, the loss associated with a lack of credibility takes the form of a long-run *inflationary bias*. Whether this is a real problem or a non-problem (as argued by Blinder (1998)) is open to question. For a dynamic models of the New Keynesian genre, such as that employed in this paper, the influential review of Clarida *et al.* (1999) emphasizes the *stabilization gain* from commitment which exist whether or not there is a long-run inflationary bias. But what is the size of this stabilization gain from commitment?

One contribution of this paper is to answer this question using a standard DSGE model estimated by Bayesian methods. In doing so we address an important consideration when addressing the gain from commitment, namely the existence of a zero lower bound for the nominal interest rate. Adam and Billi (2006) show that ignoring this constraint on the setting of the nominal interest rate can result in considerably underestimating the stabilization gain from commitment. The reason for this is that under discretion the monetary authority cannot make credible promises about future policy. For a given setting of future interest rates the volatility of inflation is driven up by the expectations of the private sector that the monetary authority will re-optimize in the future. This means that to achieve a given low volatility of inflation the lower bound is reached more often under discretion than under commitment.

The second contribution of this paper is to assess the credibility problem in an open economy context where central banks can act strategically. DSGE modelling of open economies has been a very active area in the last decade. Following from this literature we have witnessed a new look at an old issue in monetary policy: what is the potential gain from policy coordination and how can this be sustained? We provide an assessment of these developments and show how the coordination problem is closed related to the time-inconsistency problem.

The rest of the paper is organized as follows. Section 2 uses a simple New Keynesian model to show analytically how a stabilization bias arises in models with structural dynamics. Section 3 sets out a more developed New Keynesian with persistent mechanisms taking the form of habit formation in consumption and labour supply and price indexing. A linearization of the model about a zero-interest steady state and a quadratic approximation of the representative household's utility sets up the optimization problem facing the monetary authority in the required LQ framework. Section 4 using Bayesian methods to estimate the model and variants where the indexing of prices and the two forms of habit formation are suppressed in turn.

Our welfare quadratic approximation assumes that the zero-inflation steady state is close to the social optimum (the 'small distortions case' of Woodford (2003)). In section 5 we therefore assess the quality of this approximation. In doing this we examine a relatively neglected aspect of New Keynesian models that arises with the inclusion of external habit in consumption, namely that the natural rate of output and employment can actually be *above* the social optimum making the inflationary bias negative and the tax wedge, up to a point, welfare-enhancing. In section 6 we address the question of the size of the stabilization gain. Section 7 explores the open-economy aspects of the time-inconsistency problem and Section 8 concludes the paper.

2 The Time Inconsistency Problem

We first demonstrate how a stabilization bias in addition to the better known long-run inflationary bias can arise using two simple and now very standard DSGE models. The first popularized notably by Clarida *et al.* (1999) and Woodford (2003) is 'New Keynesian' and takes the form.

$$\pi_t = \beta E_t \pi_{t+1} + \lambda x_t + u_t \tag{1}$$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) \tag{2}$$

In (1) and (2), π_t is the inflation rate, β is the private sector's discount factor, $E_t(\cdot)$ is the expectations operator and x_t is output measured relative to its flexi-price value, the 'output gap', which equals consumption measured relative to its flexi-price value in this closed-economy model without capital stock or government spending. (1) is derived as a linearized form of Calvo staggered price setting about a zero-inflation steady state and (2) is a linearized Euler equation with nominal interest rate i_t and a risk aversion parameter σ. u_t is a zero-mean shock to marginal costs. All variables are expressed as deviations about the steady state, π_t and i_t as absolute deviations, and x_t as a proportional deviation.

The second model simply replaces (1) with a 'New Classical Phillips Curve' (see Woodford (2003), chapter 3):

$$\pi_t = E_{t-1}\pi_t + \lambda x_t + u_t \tag{3}$$

This aggregate supply curve can be derived by assuming some firms fix prices one period in advance and others can adjust immediately.

Kydland and Prescott (1977) and Barro and Gordon (1983) employed the 'New Classical Phillips Curve' (3) and showed that a time-inconsistency or credibility problem in monetary policy arises when the monetary authority at time 0 sets a state-contingent inflation rate π_t to minimize a loss function

$$\Omega_0 = E_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t \left[w_y (x_t - k)^2 + \pi_t^2 \right] \right] \tag{4}$$

Having set the inflation rule, the Euler equation (2) then determines the nominal interest rate that will put the economy on a path with the implied interest rate trajectory. The constant k in the loss function arises because the steady state is inefficient owing to imperfect competition and other distortions.[2] For this simple, essentially static model of the economy (it is really SGE rather than DSGE), optimal rules must take the form of a constant deterministic component plus a stochastic shock-contingent component. These rules depend on whether the policymaker can commit, or whether she exercises discretion and engages in period-by-period optimization. The standard results in these two cases are respectively:

$$\pi_t = \frac{w_y}{w_y + \lambda^2} u_t = \pi^C(u_t) \tag{5}$$

$$\pi_t = \frac{w_u k}{\lambda} + \frac{w_y}{w_y + \lambda^2} u_t = \pi^D(u_t) \tag{6}$$

Thus the optimal inflation rule with commitment, $\pi^C(u_t)$ consists of zero average inflation plus a shock-contingent component which sees inflation raised (i.e., monetary policy relaxed) in the face of a negative supply shock. The discretionary policy, $\pi^D(u_t)$, can be implemented as a rule with the *same* shock-contingent component as the ex ante optimal rule. The *only* difference now is that it includes a non-zero average inflation or *inflationary* bias equal to $\frac{w_u k}{\lambda}$ which renders the rule time-consistent. The credibility or 'time-inconsistency' problem, first raised by Kydland and Prescott, was simply how to *eliminate the inflationary bias whilst retaining the flexibility to deal with exogenous shocks.*

We have established that there are no stabilization gains from commitment in a model economy characterized by the New Classical Philips Curve. This is no the case when we move to the New Keynesian Phillips Curve, (1). Then using general optimization procedures described below in Section 6.3, (5) and (6) now become

$$\pi_t^C = \pi_t^C(u_t, u_{t-1}) = \delta \pi_{t-1}^C + \delta(u_t - u_{t-1}) \tag{7}$$

$$\pi_t^D = \pi_t^D(u_t) = \frac{w_u k}{\lambda} + \frac{w_u}{w_u + \lambda^2} u_t \tag{8}$$

where $\delta = \frac{1-\sqrt{1-4\beta b^2}}{2b\beta}$.[3] Comparing these two sets of results we see that the discretionary rule is unchanged, but the commitment rule now is a rule responding to past shocks (i.e.,

[2] We examine these sources of inefficiency further in Section 5.

[3] See also Clarida *et al.* (1999)

is a *rule with memory*) and therefore the stabilization component of the commitment rules now differs from that of the discretionary rule. Since the commitment rule is the ex ante optimal policy it follows that there are also now *stabilization gains from commitment.* The time-inconsistency problem facing the monetary authority in a New Keynesian economic environment now becomes the elimination of the inflationary bias whilst retaining the flexibility to deal with exogenous shocks *in an optimal way.*

3 A DSGE Model with Structural Dynamics

In the rest of the paper we will conduct the analysis in the context of a more developed but sill fairly standard DSGE model. Our model is essentially the influential model of Smets and Wouters (2003), but without physical capital and wage stickiness, but with a distortionary tax on income and habit formation in labour supply. There is a risk-free nominal bond. A final homogeneous good is produced competitively using a CES technology consisting of a continuum of differentiated goods. Intermediate goods producers and household suppliers of labor have monopolistic power. Nominal prices of intermediate goods are sticky. We incorporate habit formation in both consumption and labour supply. There is Calvo price setting with indexing of prices for those firms who, in a particular period, do not re-optimize their prices. Our model is stochastic with exogenous AR(1) stochastic processes for total factor productivity in the intermediate goods sector and government spending.

3.1 Households

There are ν households of which a representative household r in the home bloc maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t(r) - H_{C,t})^{1-\sigma}}{1-\sigma} + \chi \frac{\left(\frac{M_t(r)}{P_t}\right)^{1-\varphi}}{1-\varphi} - \kappa \frac{(N_t(r) - H_{N,t})^{1+\phi}}{1+\phi} + u(G_t) \right] \tag{9}$$

where E_t is the expectations operator indicating expectations formed at time t and β is the household's discount factor, $C_t(r)$ is an index of consumption, $N_t(r)$ are hours worked, $H_{C,t}$ and $H_{N,t}$ represents the habit, or desire not to differ too much from other households, and we choose $H_{C,t} = h_C C_{t-1}$, where $C_t = \frac{1}{\nu} \sum_{r=1}^{\nu} C_t(r)$ is the average consumption index, $H_{N,t} = h_N \frac{N_{t-1}}{\nu}$, where N_t is aggregate labour supply defined after (11) below and $h_C, h_N \in [0, 1)$. When $h_C = 0$, $\sigma > 1$ is the risk aversion parameter (or the inverse of the intertemporal elasticity of substitution). $M_t(r)$ are end-of-period nominal money balances and $u(G_t)$ is the utility from exogenous real government spending G_t. We normalize the household number ν at unity.

The representative household r must obey a budget constraint:

$$P_tC_t(r)+D_t(r)+M_t(r)=W_t(r)(1-T_t)N_t(r)+(1+i_{t-1})D_{t-1}(r)+M_{t-1}(r)+\Gamma_t(r) \quad (10)$$

where P_t is a price index, $D_t(r)$ are end-of-period holdings of riskless nominal bonds with nominal interest rate i_t over the interval $[t,t+1]$. $W_t(r)$ is the wage, $\Gamma_t(r)$ are dividends from ownership of firms net of any lump-sum taxes and T_t is a tax on labour income.[4] In addition, if we assume that households' labour supply is differentiated with elasticity of supply η, then (as we shall see below) the demand for each household's labour is given by

$$N_t(r)=\left(\frac{W_t(r)}{W_t}\right)^{-\eta}N_t \quad (11)$$

where $W_t=\left[\int_0^1 W_t(r)^{1-\eta}dr\right]^{\frac{1}{1-\eta}}$ is an average wage index and N_t is average employment.

Maximizing (9) subject to (10) and (11) and imposing symmetry on households (so that $C_t(r)=C_t$, etc) yields standard results:

$$1 = \beta(1+i_t)E_t\left[\left(\frac{(C_{t+1}-H_{C,t+1})^{-\sigma}}{(C_t-H_{C,t})^{-\sigma}}\right)\frac{P_t}{P_{t+1}}\right] \quad (12)$$

$$\left(\frac{M_t}{P_t}\right)^{-\varphi} = \frac{(C_t-H_{C,t})^{-\sigma}}{\chi P_t}\left[\frac{i_t}{1+i_t}\right] \quad (13)$$

$$\frac{W_t(1-T_t)}{P_t} = \frac{\kappa}{(1-\frac{1}{\eta})}(N_t-H_{N,t})^{\phi}(C_t-H_{C,t})^{\sigma} \quad (14)$$

(12) is the familiar Keynes-Ramsey rule adapted to take into account of the consumption habit. In (13), the demand for money balances depends positively on consumption relative to habit and negatively on the nominal interest rate. Given the central bank's setting of the latter, (13) is completely recursive to the rest of the system describing our macro-model. (14) equates the real post tax wage with the marginal rate of substitution between work and consumption, marked up to reflect the market power of households arising from their monopolistic supply of a differentiated factor input with elasticity η.

3.2 Firms

Competitive final goods firms use a continuum of non-traded intermediate goods according to a constant returns CES technology to produce aggregate output

$$Y_t=\left(\int_0^1 Y_t(m)^{(\zeta-1)/\zeta}dm\right)^{\zeta/(\zeta-1)} \quad (15)$$

[4]In fact as in Levine *et al.* (2006b) T_t can be interpreted as a total tax wedge.

where ζ is the elasticity of substitution. This implies a set of demand equations for each intermediate good m with price $P_t(m)$ of the form

$$Y_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} Y_t \tag{16}$$

where $P_t = \left[\int_0^1 P_t(m)^{1-\zeta} dm\right]^{\frac{1}{1-\zeta}}$. P_t is an aggregate intermediate price index, but since final goods firms are competitive and the only inputs are intermediate goods, it is also the domestic price level.

In the intermediate goods sector each good m is produced by a single firm m using only differentiated labour with another constant returns CES technology:

$$Y_t(m) = A_t \left(\int_0^1 N_t(r,m)^{(\eta-1)/\eta} dr\right)^{\eta/(\eta-1)} \tag{17}$$

where $N_t(r,m)$ is the labour input of type r by firm m and A_t is an exogenous shock capturing shifts to trend total factor productivity (TFP) in this sector. Minimizing costs $\int_0^1 W_t(r)N_t(r,m)dr$ and aggregating over firms and denoting $\int_0^1 N_t(r,m)dm = N_t(r)$ leads to the demand for labor as shown in (11). In an equilibrium of equal households and firms, all wages adjust to the same level W_t and it follows that $Y_t = A_t N_t$.

For later analysis it is useful to define the real marginal cost as the wage relative to domestic producer price. Using (14) and $Y_t = A_t N_t$ this can be written as

$$MC_t \equiv \frac{W_t}{A_t P_t} = \frac{1}{(1-\frac{1}{\eta})(1-T_t)A_t} (N_t - H_{N,t})^{\phi} (C_t - H_{C,t})^{\sigma} \tag{18}$$

Now we assume that there is a probability of $1-\xi$ at each period that the price of each intermediate good m is set optimally to $P_t^0(m)$. If the price is not re-optimized, then it is indexed to last period's aggregate producer price inflation.[5] With indexation parameter $\gamma \geq 0$, this implies that successive prices with no re-optimization are given by $P_t^0(m)$, $P_t^0(m)\left(\frac{P_t}{P_{t-1}}\right)^{\gamma}$, $P_t^0(m)\left(\frac{P_{t+1}}{P_{t-1}}\right)^{\gamma}$, ... For each intermediate producer m the objective is at time t to choose $\{P_t^0(m)\}$ to maximize discounted profits

$$E_t \sum_{k=0}^{\infty} \xi^k Q_{t+k} Y_{t+k}(m) \left[P_t^0(m)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma} - \frac{W_{t+k}}{A_{t+k}}\right] \tag{19}$$

given i_t (since firms are atomistic), subject to (16), where Q_{t+k} is the discount factor over the interval $[t, t+k]$. The solution to this is

$$E_t \sum_{k=0}^{\infty} \xi^k Q_{t+k} Y_{t+k}(m) \left[P_t^0(m)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma} - \frac{1}{(1-1/\zeta)} \frac{W_{t+k}}{A_{t+k}}\right] = 0 \tag{20}$$

[5] Thus we can interpret $\frac{1}{1-\xi}$ as the average duration for which prices are left unchanged.

and by the law of large numbers the evolution of the price index is given by

$$P_{t+1}^{1-\zeta} = \xi \left(P_t \left(\frac{P_t}{P_{t-1}} \right)^{\gamma} \right)^{1-\zeta} + (1-\xi)(P_{t+1}^0)^{1-\zeta} \tag{21}$$

3.3 Equilibrium

In equilibrium, goods markets, money markets and the bond market all clear. Equating the supply and demand of the consumer good we obtain

$$Y_t = A_t N_t = C_t + G_t \tag{22}$$

Assuming the same tax rate levied on all income (wage income plus dividends) a balanced budget government budget constraint

$$P_t G_t = P_t T_t Y_t + M_t - M_{t-1} \tag{23}$$

completes the model. As in Coenen *et al.* (2005) we further assume that changes in government spending are financed exclusively by changes in lump-sum taxes with the tax rate T_t held constant at its steady-state value. Given interest rates i_t, expressed later either in terms of an optimal or an Inflation Forecast-Based (IFB) rule, the money supply is fixed by the central banks to accommodate money demand. By Walras' Law we can dispense with the bond market equilibrium condition and therefore the government budget constraint that determines taxes τ_t. Then the equilibrium is defined at $t = 0$ by stochastic processes C_t, D_t, P_t, M_t, W_t, Y_t, N_t, given past price indices and exogenous TFP and government spending processes.

In what follows we will assume a 'cashless economy' version of the model in which both seigniorage in (23) and the utility contribution of money balances in (9) are negligible. Then given the nominal interest rate, our chosen monetary instrument, we can dispense altogether with the money demand relationship (13).

3.4 Zero-Inflation Steady State

A deterministic zero-inflation steady state, denoted by variables without the time subscripts, is given by

$$\begin{aligned}
\frac{W(1-T)}{P} &= \frac{\kappa(1-h_N)^{\phi}(1-h_C)^{\sigma}}{1-\frac{1}{\eta}} N^{\phi} C^{\sigma} && (24)\\
1 &= \beta(1+i) && (25)\\
Y &= AN && (26)\\
P &= P^0 = \frac{W}{A\left(1-\frac{1}{\zeta}\right)} && (27)\\
Y &= C+G && (28)\\
T &= \frac{G}{Y} && (29)
\end{aligned}$$

giving us in effect 7 equations to determine $\frac{W}{P}$, i, C, N, Y, $\frac{P}{P0}$ and T. The natural rate of interest is determined by the private sector's discount factor. In our cashless economy the price level is indeterminate.

4 Bayesian Estimation

In this section we conduct a Bayesian estimation of the linearized form, about the steady state, of the model as in Batini *et al.* (2004). We estimate the following model variants:

Model 1: $h_C = h_N = 0$.
Model 2: $h_C > 0$, $h_L = \gamma = 0$
Model 3: $h_C = 0$, $h_L > 0$, $\gamma = 0$
Model 4: $h_C = 0$, $h_L = 0$, $\gamma > 0$
Model 5: $h_C > 0$, $h_L > 0$, $\gamma = 0$
Model 6: $h_C > 0$, $h_L = 0$, $\gamma > 0$
Model 7: $h_C = 0$, $h_L > 0$, $\gamma > 0$
Model 8: $h_C > 0$, $h_L > 0$, $\gamma > 0$

Three observables in the estimated model variants are output, annualized inflation and annualized Fed Funds rate series for the US. Since the variables in the model are measured as deviations from a constant steady state, the data is simply detrended against a linear trend. The estimation results are based on a sample from 1983:1 to 2002:4 and 8 observations are used to initialize the Kalman filter. Moreover, two of the structure parameters are kept fixed in the following estimation procedure. Suggested by Adolfson *et al.* (2005), these parameters can be related to the steady state values of the observed variables in the model, and are therefore calibrated so as to match the sample mean of these. The discount factor β is set to 0.99, which implies an annual steady state nominal interest rate of 5.5 percent. A common theme in papers estimating DSGE models is the difficulty in pinning down the parameter of labour supply elasticity ϕ, inference on the inverse Frisch elasticity of labour supply has been found susceptible to model specifications, and exhibiting wide posterior probability intervals.(Batini *et al.* (2004)) As a result, following Christiano *et al.* (2005), the parameter ϕ is set to unity. They also argue that although this calibrated elasticity is low by comparison with the values assumed in the real business cycle literature, it is well within the range of point estimates reported in the labour literature. (See, for instance, Rotemberg and Woodford (1999))

All analysis is performed with the DYNARE (Matlab version) programme (Juillard (2006))[6] and Matlab. The IFB policy rule contained in the models is the one-quarter ahead forward-looking rule and we include estimation of the interest rate smoothing parameter. In deviation form this is given by

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\theta E_t \pi_{t+1} \tag{30}$$

In order to avoid stochastic singularity when evaluating the likelihood function, Dynare requires at least as many shocks or measurement errors in the models as observable variables (i.e. requires that the covariance matrix of endogenous variables is nonsingular). In this estimation an additional structural shock is included to capture to some extent aggregation effects (e.g. monetary policy shock) and there is no measurement error in the data set without facing stochastic singularity. The mode of the posterior is first estimated using the MATLAB's fmincon (and Chris Sim's csminwel[7]) after the models' log-prior densities and log-likelihood functions have been obtained by running the Kalman recursion and maximized. Then a sample from the posterior distribution is obtained with the Metropolis-Hastings (MH) algorithm using the inverse Hessian at the estimated posterior mode as the covariance matrix of the jumping distribution. The covariance matrix needs to be adjusted in order to obtain reasonable acceptance rates. Thus the scale used for the jumping distribution in the MH is set to 0.25, allowing good acceptance rates (around 40%-60%). Two parallel Markov chains of 100000 runs each are run from the posterior kernel for the MH. The first 25% of iterations (initial burn-in period) are discarded in order to remove any dependence of the chain from its starting values. The estimation results

[6]Version 3.064 of the package is available for downloading from the Dynare homepage: http://www.cepremap.cnrs.fr/juillard/mambo/index.php

[7]See, for more details, Chris Sim's homepage: http://www.princeton.edu/ sims/

then report the Bayesian inference. (Table 2 below summarises the prior distribution, posterior mean and median and 90% confidence interval for the eight model specifications). The posterior median is calculated by sorting the draws from the marginal distribution of the parameters and computing the value of the median after the MCMC is finished. The marginal data density of each model is computed using the Geweke (1999) modified harmonic-mean estimator.

As shown in Table 2 in the annex, estimates of the policy coefficients are fairly robust across specifications. As expected, the policy rule estimates imply a fairly strong response (θ) to expected inflation by the US Fed Reserve and the degree of interest rate smoothing (ρ_i) is substantial. All shocks from all the model variants are found fairly persistent and volatile except that the technology shock seems to be less persistent in the models without any habit formation (i.e. Models 1, 3, 4 and 7). As usual, monetary policy disturbances (ε_e) are less important in driving inflation, consumption and output. As also discussed in Batini *et al.* (2004), the estimates of γ imply that inflation is intrinsically not very persistent in the relevant model specifications. The estimated mean and median values of around 0.995 for the stochastic discount factors are very close to the conventional calibrated value of β. On the demand side, it is found that both habit formations, especially consumption habit, seem to play an important role in the US economy. In addition, the risk-aversion parameter (σ) is very small when consumption habit is absent, indicating that the intertemporal elasticity of substitution (proportional to $1/\sigma$) may be quite large for Models 1, 3, 4 and 7. For Models 6 and 8 , the larger value of the slope of the Phillips curve (λ) corresponds to a smaller ξ which indicates that nominal rigidity and inertia in the price settings seems to be reduced. The median estimates for the real interest rate i^* translate into a median value of around 0.995 for the stochastic discount factor which, in turn, implies plausible estimates for the degree of price stickiness based on the inferred values for the Phillips curve slope λ. Finally, the mean/median estimates of β, γ, λ determine the point estimates for the degree of price stickiness ξ, which is then found to be fairly strong and in accordance with the values estimated by Blinder *et al.* (1998) and Rotemberg and Woodford (1998). In particular, ξ ranges from 0.41 up to 0.71 , corresponding to contract lengths, 3.14, 2.80, 3.45, 2.54, 3.23, 1.69, 2.66 and 1.87 quarters for Models 1-8, respectively, which seem to be close to the plausible estimates [8].

The problem of Bayesian model comparisons is to use data to determine which of the eight competing models is closer to the 'truth'. We compare the posterior model probabilities given data, $P(M/D)$, which is given by Bayes' theorem: $P(M/D) = P(D/M)P(M)/P(D)$. The key data-dependent term $P(D/M)$ is the marginal data density, which is produced

[8] ξ is obtained by using $\lambda \equiv \frac{(1-\beta\xi)(1-\xi)}{(1+\beta\gamma)\xi}$; average contract length $\frac{1}{1-\xi}$ is measured in quarters.

by running DYNARE. Given that the prior probability of each model is assigned equal weight, $P(M_k/D) \propto P(D/M_k) = \exp^{LL_k}$. The posterior odds ratio then satisfies:

$$\frac{P(M_j/D)}{P(M_i/D)} \propto \frac{P(D/M_j)}{P(D/M_i)} = \frac{\exp^{LL_j}}{\exp^{LL_i}} \tag{31}$$

and is normalized to $P(M_j/D)/\sum_i P(M_i/D)$ This is the bottom line of Table 2 which indicates that Model 2 (with consumption habit but no labour habit or price indexation) outperforms its 7 rivals. However the performance when including labour habit persistence in improving model fit appears ambiguous to interpret. On the other hand, the second most restrictive model (Model 4, with only price indexation) seems to be worst supported by the data. These results clearly suggest that incorporating habit persistence in consumption in the US model imparts greater inertia to the model, and improves the fit. In the policy analysis of Section 6 we have used the median parameter estimates of Model 2.

5 The Inflation Bias and Optimal Taxation

The natural rate of output is below the efficient rate because of monopoly power in output and labour markets, and external habit in labour supply. However, external habit in consumption works in the opposite direction. To see this we solve for the deterministic social planner's problem.[9] Using (9) the social planner chooses trajectories for output and inflation to maximize

$$\Omega_0 = \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - h_C C_{t-1})^{1-\sigma}}{1-\sigma} - \kappa \frac{(N_t - h_N N_{t-1})^{1+\phi}}{(1+\phi)} \right] \tag{32}$$

where $C_t = Y_t - G_t$ and $N_t = \frac{Y_t}{A_t}$. The first-order condition for the choice out output is

$$[C_t - h_C C_{t-1}]^{-\sigma} - h_C \beta [C_{t+1} - h_C C_t]^{-\sigma} = \frac{\kappa}{A_t} \left[\left[\frac{Y_t}{A_t} - h_N \frac{Y_{t-1}}{A_{t-1}} \right]^{\phi} - h_N \beta \left[\frac{Y_{t+1}}{A_{t+1}} - h_N \frac{Y_t}{A_t} \right]^{\phi} \right] \tag{33}$$

The efficient steady-state level of output $Y_{t+1} = Y_t = Y_{t-1} = Y^*$, say, is therefore given by

$$(Y^*)^{\phi} (Y^* - G)^{\sigma} = \frac{(1 - h_C \beta) A^{1+\phi}}{\kappa (1 - h_N \beta)(1 - h_C)^{\sigma} (1 - h_N)^{\phi}} \tag{34}$$

From (24) to (28), after some manipulation, the steady-state level of output (the 'natural rate'), is given by

$$Y^{\phi} (Y - G)^{\sigma} = \frac{(1 - T) \left(1 - \frac{1}{\zeta}\right) \left(1 - \frac{1}{\eta}\right) A^{1+\phi}}{\kappa (1 - h_C)^{\sigma} (1 - h_N)^{\phi}} \tag{35}$$

[9]We assume zero inflation and therefore no welfare costs from the dispersion of labour demand across firms. We return to inflation costs from this source later.

Comparing (35) and (34), since $(Y)^\phi(Y-G)^\sigma$ is an increasing function of Y, we arrive at[10]

Proposition

The natural level of output, Y, is below the efficient level, Y^*, if and only if

$$(1-T)\left(1-\frac{1}{\zeta}\right)\left(1-\frac{1}{\eta}\right) < \frac{1-h_C\beta}{1-h_N\beta} \tag{36}$$

In the case where there is no habit persistence for both consumption and labour effort, $h_C = h_N = 0$, then (36) always holds. In this case tax distortions and market power in the output and labour markets captured by the elasticities $\eta, \zeta \in (1, \infty)$ drive the natural rate of output below the efficient level. If $T = 0$ and $\eta = \zeta = \infty$, tax distortions and market power disappear and the natural rate is efficient. Another case when (36) always holds is where habit persistence for labour supply exceeds that for consumption; i.e., $h_N \geq h_C$. Our empirical estimates strongly suggest that $h_C > h_N$ which leads to the possibility that the natural rate of output can actually be *above* the efficient level.

To pursue this possibility there are two remaining parameters η and ζ to calibrate.[11] The mark-up of the real wage disposable wage on the marginal rate of substitution and the mark-up of the price on the marginal cost are given by $\frac{1}{1-\frac{1}{\eta}}$ and $\frac{1}{1-\frac{1}{\zeta}}$ respectively. Suppose we set these mark-ups as equal and defined by μ as one or other of 1.10, 1.15, 1.20 and 1.30. Then the optimal tax wedge that will equate the natural rate and socially optimal output levels in the steady state is given by

$$T^* = 1 - \frac{(1-h_C\beta)\mu^2}{(1-h_N\beta)} \tag{37}$$

and for our favoured Model 2 with $h_C = 0.86$, $h_N = 0$ this takes values $T^* = 0.82$, 0.80, 0.77 and 0.75. These are very large tax wedges, much higher than those in the US and the Euro Area.[12] Our conclusion is that given our estimates for Model 2, from a welfare perspective, the tax wedge is 'corrective' (in the words of Layard (2006)), rather than distortionary, and that the existing wedge in the US is too low. The further implication is that the constant k in (4) is negative, so if we are to accept the Barro-Gordon argument, *the inflationary bias is negative*! In view of these findings and the 'Blinder Critique' (Blinder (1998)) of the inflation bias, in what follows we focus solely on the stabilization gain from commitment in a set-up where the policymaker does not try to compensate for an inefficient steady state natural rate of output.

[10] See Choudhary and Levine (2006).

[11] An examination of the linearized form of the models reveals the fact that η and ζ are not identified.

[12] Coenen *et al.* (2005) report total tax wedges of 37.3% and 64.1% respectively for the US and the Euro Area.

6 Optimal Monetary Policy

6.1 The Stabilization Bias in General DSGE Models

The stabilization bias arose in our simple DSGE model by replacing a Phillips Curve based on one-period ahead price contracts with one based on Staggered Calvo-type price setting. The more developed DSGE model presented above added *structural dynamics* to the model and can be written in the following linear state-space form:

$$\begin{bmatrix} \mathsf{z}_{t+1} \\ E_t \mathsf{x}_{t+1} \end{bmatrix} = A \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} + B i_t + C \epsilon_{t+1} \tag{38}$$

$$\mathsf{o}_t = E \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} \tag{39}$$

where z_t is a $(n-m) \times 1$ vector of predetermined variables at time t with z_0 given, x_t, is a $m \times 1$ vector of non-predetermined variables and o_t is a vector of outputs. A, B, C and E are fixed matrices and ϵ_t is a vector of random zero-mean shocks. Rational expectations are formed assuming an information set $\{z_s, x_s, \epsilon_s\}$, $s \leq t$, the model and the monetary rule. z_t consists of exogenous shocks, and lagged variables; x_t consists of current inflation and consumption. x_t also includes flexi-price outcomes for the latter two variables, and outputs o_t consist of marginal costs, the marginal rate of substitution for consumption and leisure, labour supply, output, flexi-price outcomes, the output gap and other target variables for the monetary authority. Let $\mathsf{s}_t = M[\mathsf{z}_t^T \mathsf{x}_t^T]^T$ be the vector of such target variables. For both the ad hoc and welfare-based loss function discussed below, the inter-temporal loss function (4) generalizes to

$$\Omega_0 = E_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t L_t \right] \tag{40}$$

where the single-period loss function is given by $L_t = \mathsf{s}_t^T Q_1 \mathsf{s}_t = \mathsf{y}_t^T Q \mathsf{y}_t$ where $\mathsf{y}_t = [\mathsf{z}_t^T \mathsf{x}_t^T]^T$ and $Q = M^T Q_1 M$.

6.2 Imposing a Lower Interest Rate Bound Constraint

In the absence of a zero lower bound (henceforth ZLB) constraint on the nominal interest rate the policymaker's optimization problem is to minimize (40) subject to (A.11) and (A.12). Then complete stabilization of the output gap and inflation is possible, but if shock variances are sufficiently large this will lead to a large variability of the nominal interest rate and the possibility of it becoming negative. To rule out this possibility and to remain within the convenient LQ framework of this paper, we follow Woodford (2003), chapter 6, and approximate the ZLB effect by introducing constraints of the form

$$E_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t i_t \right] \geq 0 \tag{41}$$

$$E_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t i_t^2 \right] \leq K \left[E_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t i_t \right] \right]^2 \tag{42}$$

Woodford shows that the effect of these extra constraints is to follow the same optimization as before except that the single period loss function is replaced with

$$L_t = \mathsf{y}_t^T Q \mathsf{y}_t + w_i (i_t - i^*)^2 \tag{43}$$

where $w_i > 0$ if (42) binds (which we assume) and $i^* > 0$ if monetary transactions frictions are negligible, but $i^* < 0$ is possible otherwise (i.e., the interest rate must be lower than that necessary to keep inflation zero in the steady state). In what follows we put $i^* = 0$.

6.3 Commitment Versus Discretion

To derive the ex ante optimal policy with commitment following Currie and Levine (1993) we maximize the the Lagrangian

$$\mathcal{L}_0 = E_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t \left[\mathsf{y}_t^T Q \mathsf{y}_t + w_i i_t^2) + \mathsf{p}_{t+1}(A\mathsf{y}_t + Bi_t - \mathsf{y}_{t+1}) \right] \right] \tag{44}$$

with respect to $\{i_t\}$, $\{\mathsf{y}_t\}$ and the row vector of costate variables, p_t, given z_0. From Appendix A of Levine *et al.* (2006b) where more details are provided, this leads to a optimal rule of the form

$$i_t = D \begin{bmatrix} \mathsf{z}_t \\ \mathsf{p}_{2t} \end{bmatrix} \tag{45}$$

where

$$\begin{bmatrix} \mathsf{z}_{t+1} \\ \mathsf{p}_{2t+1} \end{bmatrix} = H \begin{bmatrix} \mathsf{z}_t \\ \mathsf{p}_{2t} \end{bmatrix} \tag{46}$$

and the optimality condition[13] at time $t = 0$ imposes $\mathsf{p}_{20} = 0$. In (45) and (46) $\mathsf{p}_t^T = \left[\mathsf{p}_{1t}^T \, \mathsf{p}_{2t}^T\right]$ is partitioned so that p_{1t}, the co-state vector associated with the predetermined variables, is of dimension $(n-m)$ and p_{2t}, the co-state vector associated with the non-predetermined variables, is of dimension m. The loss function is given by

$$\Omega_t^{OP} = -(1-\beta)\mathrm{tr}\left(N_{11} \left(Z_t + \frac{\beta}{1-\beta}\Sigma \right) + N_{22}\mathsf{p}_{2t}\mathsf{p}_{2t}^T \right) \tag{47}$$

[13]Optimality from a 'timeless perspective' imposes a different condition at time $t = 0$ (see Appendix A.1.2), but this has no bearing on the stochastic component of policy, the focus of this paper.

where $Z_t = \mathsf{z}_t\mathsf{z}_t^T$, $\Sigma = \text{cov}(\epsilon_t)$,

$$N = \begin{bmatrix} S_{11} - S_{12}S_{22}^{-1}S_{21} & S_{12}S_{22}^{-1} \\ -S_{22}^{-1}S_{21} & S_{22}^{-1} \end{bmatrix} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \tag{48}$$

and S is the solution to the steady-state Ricatti equation. In (48) matrices S and N are partitioned conformably with $\mathsf{y}_t = [\mathsf{z}_t^T \mathsf{x}_t^T]^T$ so that S_{11} for instance has dimensions $(n-m) \times (n-m)$.

Note that in order to achieve optimality the policy-maker sets $\mathsf{p}_{20} = 0$ at time $t = 0$. At time $t > 0$ there exists a gain from reneging by resetting $\mathsf{p}_{2t} = 0$. It can be shown that N_{22} is negative definite, so the incentive to renege exists at all points along the trajectory of the optimal policy. This essentially is the time-inconsistency problem facing stabilization policy in a model with structural dynamics.

To evaluate the discretionary (time-consistent) policy we write the expected loss Ω_t at time t as

$$\Omega_t = E_t\left[(1-\beta)\sum_{\tau=t}^{\infty}\beta^{\tau-t}L_\tau\right] = (1-\beta)(\mathsf{y}_t^T Q\mathsf{y}_t + w_i i_t^2) + \beta\Omega_{t+1} \tag{49}$$

The dynamic programming solution then seeks a stationary solution of the form $i_t = -Fz_t$, $\Omega_t = \mathsf{z}^T S\mathsf{z}$ and $\mathsf{x} = -N\mathsf{z}$ where matrices S and N (completely unrelated to those defined for the commitment case) are now of dimension $(n-m) \times (n-m)$ and $m \times (n-m)$ respectively, in which Ω_t is minimized at time t subject to (1) in the knowledge that a similar procedure will be used to minimize Ω_{t+1} at time $t+1$.[14] Both the instrument i_t and the forward-looking variables x_t are now proportional to the predetermined component of the state-vector z_t and the equilibrium we seek is therefore *Markov Perfect*. We can set this out as an iterative process for F_t, N_t, and S_t starting with some initial values. If the process converges to stationary values independent of these initial values,[15] F, N and S say, then the time-consistent feedback rule is $i_t = -Fz_t$ with loss at time t given by

$$\Omega_t^{TC} = (1-\beta)\text{tr}\left(S\left(Z_t + \frac{\beta}{1-\beta}\Sigma\right)\right) \tag{50}$$

6.4 Formulating the Policymaker's Loss Function

Much of the optimal monetary policy literature has stayed with the ad hoc loss function (4) which, with a interest rate lower bound constraint, becomes

$$\Omega_0 = E_0\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t\left[(y_t - \hat{y}_t - k)^2 + w_\pi\pi_t^2 + w_i i_t^2\right]\right] \tag{51}$$

[14] See Currie and Levine (1993) and Söderlind (1999).

[15] Indeed we find this is the case in the results reported in the paper.

Indeed Clarida *et al.* (1999) provide a stout defence of a hybrid research strategy that combines a loss function based on the stated objectives of central banks with a micro-founded macro-model. A normative assessment of policy rules requires welfare analysis and for this, given our linear-quadratic framework,[16] we require a quadratic approximation of the representative consumer's utility function.

A common procedure for reducing optimal policy to a LQ problem is as follows. Linearize the model about a deterministic zero-inflation steady state as we have already done. Then expand the consumer's utility function as a second-order Taylor series after imposing the economy's resource constraint. In general this procedure is incorrect unless the steady state is not too far from the efficient outcome (see Woodford (2003), chapter 6, Benigno and Woodford (2004), Kim and Kim (2006) and Levine *et al.* (2006a)). Let us assume this, and for this case we show in Levine *et al.* (2006b) that a quadratic single-period loss function that approximates the utility takes the form

$$L = w_c(c_t - h_C c_{t-1})^2 + w_l(l_t - h_L l_{t-1})^2 + w_\pi(\pi_t - \gamma\pi_{t-1})^2 + w_{al} a_t l_t \tag{52}$$

where positive weights w_c etc are defined as follows:

$$w_c = \sigma\,;\ w_l = \frac{\phi}{c_y}\,;\ w_\pi = \frac{\zeta\xi}{(1-\xi)(1-\beta\xi)}\,;\ w_{al} = -2 \tag{53}$$

where c_y is the steady state ratio C/Y. All variables are in log-deviation form about the steady state as in the linearization. When there is no habit ($h_C = h_L = 0$) or government spending ($c_y = 1$), $c_t = y_t = l_t - a_t$ and we end up with the loss function in Woodford (2003):

$$\Omega_0 = E_0\left[\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[(\sigma+\phi)(y_t - \hat{y}_t)^2 + w_\pi(\pi_t - \gamma\pi_{t-1})^2\right]\right] \tag{54}$$

where $\hat{y}_t = \frac{1+\phi}{\sigma+\phi}a_t$ is potential output achieved when prices are flexible. By suitably redefining w_π in (51), the micro-founded welfare-based loss function (54) is of exactly the same form, give or take terms independent of policy.

From the results of this sub-section, we conclude that the traditional 'ad hoc' form of loss function *can* be reconciled with the household's welfare if the steady state is not too far from the efficient outcome, but only for a very simple DSGE model that ignores habit and government spending. In the absence of these simplifications the more general form set out in Levine *et al.* (2006a) should be used for the analysis of optimal policy from a welfare point of view.

[16] We have emphasized the convenience of the LQ approach to optimal policy. However, recent developments in numerical methods now allow the researcher to go beyond linear approximations of their models and to conduct analysis of both the dynamics and welfare using higher-order (usually second-order) approximations (see, Kim *et al.* (2003) and for an application to simple monetary policy rules, Juillard *et al.* (2004)).

6.5 Results

From our discussion of the interest rate ZLB effect in section 6.2, the policymaker's optimization problem is to choose an unconditional distribution for i_t (characterized by the steady state variance) shifted to the right about a new non-zero steady state inflation rate and a higher nominal interest rate, such that the probability, p, of the interest rate hitting the lower bound is very low. This is implemented by calibrating the weight w_i *for each of our policy rules* so that $z_0(p)\sigma_i < i$ where $z_0(p)$ is the critical value of a standard normally distributed variable Z such that prob $(Z \leq z_0) = p$, $i = \frac{1}{\beta} - 1 + \pi^*$ is the steady state nominal interest rate, σ_i is the unconditional variance and π^* is the new steady state inflation rate. Given σ_i the steady state positive inflation rate that will ensure $i_t \geq 0$ with probability $1 - p$ is given by[17]

$$\pi^* = \max[z_0(p)\sigma_i - \left(\frac{1}{\beta} - 1\right) \times 100, 0] \tag{55}$$

In our linear-quadratic framework we can write the intertemporal expected welfare loss at time $t = 0$ as the sum of stochastic and deterministic components, $\Omega_0 = \tilde{\Omega}_0 + \bar{\Omega}_0$. By increasing w_i we can lower σ_i thereby decreasing π^* and reducing the deterministic component, but at the expense of increasing the stochastic component of the welfare loss. By exploiting this trade-off, we then arrive at the optimal policy that, in the vicinity of the steady state, imposes the zero lower bound constraint, $i_t \geq 0$ with probability $1 - p$.

Table 3 shows the results of this optimization procedure under commitment using the loss function given by (52) and based on parameter estimates for our favoured model 2. We choose $p = 0.025$. Given w_i, denote the expected inter-temporal loss (stochastic plus deterministic components) at time $t = 0$ by $\Omega_0(w_i)$. This includes a term penalizing the variance of the interest rate which does not contribute to utility loss as such, but rather represents the interest rate lower bound constraint. *Actual* utility, found by subtracting the interest rate term, is given by $\Omega_0(0)$. The steady state inflation rate, π^*, that will ensure the lower bound is reached only with probability $p = 0.025$ is computed using (55). Given π^*, we can then evaluate the deterministic component of the welfare loss, $\bar{\Omega}_0$. Since in the new steady state the real interest rate is unchanged, the steady state involving real

[17] If the inefficiency of the steady-state output is negligible, then $\pi^* \geq 0$ is a credible the new steady state inflation rate. It contrasts with a transitional *deflationary bias* highlighted by Krugman (1998), Eggertsson (2006) and Adam and Billi (2006) which arises under discretion because the central bank cannot credibly lower the expected real interest rate, following a negative demand shock, by a promise to raise the inflation rate in the future. It must therefore rely on lowering the interest rate, hitting the zero lower bound more often. Reduced inflationary expectations, in turn, causes a temporary negative inflation bias. This effect is absent in the approximate approach to imposing the constraint in this paper.

variables is also unchanged, so from (52) we can write[18]

$$\bar{\Omega}_0(0) = w_\pi(1-\gamma)^2\pi^{*2} \tag{56}$$

w_i	σ_i^2	$\tilde{\Omega}_0(w_i)$	$\tilde{\Omega}_0(0)$	π^*	$\bar{\Omega}_0(0)$	$\Omega_0(0)$
0	1.57	10.4	10.4	1.46	31.1	41.5
1	0.79	11.4	10.6	0.73	7.8	18.4
2	0.62	12.1	10.8	0.53	4.1	14.9
3	0.52	12.8	11.0	0.40	2.3	13.3
4	0.45	13.3	11.2	0.30	1.3	12.5
5	0.39	13.9	11.4	0.21	0.6	12.0
6	0.35	14.3	11.6	0.15	0.33	11.93
7	**0.31**	**14.8**	**11.8**	**0.08**	**0.09**	**11.89**
8	0.28	15.2	12.0	0.03	0.01	12.01
9	0.26	15.6	12.1	0.0	0.0	12.1
10	0.23	16.0	12.3	0.0	0.0	12.3

Table 3. Optimal Commitment with a Nominal Interest Rate ZLB.

Notation: $\pi^* = \max[z_0(p)\sigma_i - (\frac{1}{\beta}-1)\times 100, 0] = \max[1.96\sigma_i - 1.01, 0]$ with $p = 2.5\%$ probability of hitting the zero-lower bound and $\beta = 0.99$. $\bar{\Omega}_0(0) = \frac{1}{2}w_\pi(1-\gamma)^2\pi^{*2} = 14.6\pi^{*2}$. $\Omega_0(0) = \tilde{\Omega}_0(0) + \bar{\Omega}_0(0)$.

Table 3 demonstrates the crucial role of the ZLB interest rate constraint in that it results in a *trade-off* between reducing the stochastic component of policy at the expense of a higher steady state inflation rate and, therefore, a higher deterministic component of policy. In the absence of the ZLB constraint, the policymaker would not need to penalize the variability of i_t and would optimize with $w_i = 0$, achieving the minimum stochastic welfare loss of $\tilde{\Omega}_0(0) = 10.4$ and a zero-inflation rate steady state. But this policy results in an unconditional steady state variance of the interest rate of $\sigma_i^2 = 1.57$ with a resulting high probability of hitting the ZLB. To reduce this probability to 2.5%, optimal policy with $w_i = 0$ must guide the economy to a non-zero inflation steady state of $\pi^* = 1.46\%$ per quarter with a corresponding large non-zero deterministic welfare loss. As the weight

[18] Both the ex-ante optimal and the optimal time-consistent deterministic welfare loss that guide the economy from a zero-inflation steady state to $\pi = \pi^*$ differ from $\bar{\Omega}_0(0)$ (but not by much because the steady state contributes by far outweighs the transitional contribution). From a *timeless perspective* (see Woodford (2003), however, the policymaker will jump immediately to the new steady state justifying the use of (56).

w_i increases, the steady state inflation rate falls at the expense of a higher stochastic component of the welfare until at $w_i = 7$, highlighted in bold, we reach the *optimal choice of* w_i that satisfies the ZLB constraint that a zero interest rate is reached with probability 2.5%.

w_i	σ_i^2	$\tilde{\Omega}_0(w_i)$	$\tilde{\Omega}_0(0)$	π^*	$\bar{\Omega}_0(0)$	$\Omega_0(0)$	No. Iters.
0	**1.11**	**11.0**	**11.0**	**1.06**	**16.4**	**27.4**	**60**
1	1.73	13.9	12.4	1.57	36.0	48.4	116
2	2.60	22.5	18.9	2.15	67.5	86.4	158
3	3.80	39.2	32.1	2.81	115.3	147.4	191
4	5.41	66.7	54.3	3.55	184.0	238.3	220
5	7.53	108.1	87.6	4.37	278.8	366.4	262
6	10.3	167.0	135.8	5.28	407.0	542.8	383
7	13.8	247.3	197.6	6.27	574.0	771.6	620
8	18.1	352.5	278.8	7.33	784.4	1063	1000
9	∞	∞	∞	∞	∞	∞	non-conv
10	∞	∞	∞	∞	∞	∞	non-conv

Table 4. Optimal Discretion with a Nominal Interest Rate ZLB.

Notation: As for table 3. 'No.Iters.' indicates the number of iterations to achieve convergence to the optimal discretionary solution.

Table 4 performs a similar exercise for optimal discretionary policy. Note that with $w_i = 0$, the unconditional variance , σ_i^2, under discretion is lower than that under commitment. To achieve the ZLB constraint then requires a larger steady state inflation under commitment than under discretion and as a result the total welfare loss is actually higher. However whereas under commitment the trade-off between a high steady-state inflation rate and a smaller stochastic welfare loss can be exploited to drastically reduce the ultimate loss, this is not the case under discretion and highlights an important difference between stabilization policy under commitment and discretion. For the latter we see that the *steady-state inflation – stochastic welfare loss trade-off now vanishes.*[19] As the weight on interest rate variability, w_i, increases, both the unconditional variance of the interest rate, and the steady-state inflation rate needed to reduce the probability of hitting the ZLB to 2.5% increase, with the consequence that $w_i = 0$ is now optimal. This is a somewhat counterintuitive result that can be explained in general by the fact that

[19]It should be stressed that this is a model-specific result. In Levine *et al.* (2006b), in a model with capital the trade-off seen under commitment re-emerges.

under discretion, a policymaker lacks the leverage to manage the economy she would enjoy under commitment. More specifically, the constraint on using the interest rate, captured by increasing the weight w_i, simply results in a more volatile economy and, in equilibrium, both the variance of the inflation rate and that of the interest rate increase.

Now we can now assess the stabilization gains from commitment. Denote the expected inter-temporal utility loss at time $t = 0$ under the time-consistent discretionary policy and optimal commitment by $\Omega_0^D(0)$ and $\Omega_0^C(0)$ respectively. We compute these gains as equivalent permanent percentage increases in consumption and inflation, c_e^{gain} and π_e^{gain} respectively. From Appendix C of Levine *et al.* (2006b), these are given by

$$c_e = \frac{\Omega_0^D(0) - \Omega_0^C(0)}{1 - h_C} \times 10^{-2}\,; \quad \pi_e = \sqrt{\frac{2(\Omega_0^D(0) - \Omega_0^C(0))}{w_\pi}} \tag{57}$$

In (57) in the absence of a ZLB constraint, we take $\Omega_0^C(0) = \tilde{\Omega}_0^C(0) = 10.4$ and $\Omega_0^D(0) = \tilde{\Omega}_0^D(0) = 11.0$ from the first rows of tables 3 and 4 respectively. With a ZLB constraint we take $\Omega_0^C(0) = 11.87$ from the $w_i = 7$ row of table 3 and $\Omega_0^D(0) = 27.4$ from the $w_i = 0$ row of table 4.

Constraint	(π^{*D}, π^{*C})	c_e^{gain}	π_e^{gain}
No ZLB Constraint	$(0, 0)$	0.043	0.041
ZLB Constraint	$(1.06, 0.08)$	1.11	1.07

Table 5. Core Model: Stabilization Gains From Commitment: % Consumption Equivalent (c_e^{gain} and % Inflation Equivalent (π_e^{gain})

Using these results table 5 summarizes the gains from commitment measured by (57) with and without interest rate ZLB considerations. In the latter case these gains are very small – of the order of a 0.04% consumption equivalent gain. It is of interest to note here that this is close to the gains from stabilization per reported by Lucas (1987). In our model these gains can be found from the minimum welfare costs under commitment. Corresponding to (57) these are $\frac{\Omega_0^C(0)}{1-h_C}$ which from our results amounts to a 0.8% consumption equivalent increase.[20]

Introducing the nominal interest rate ZLB constraint sees these stabilization gains from commitment increasing substantially to over a 1% consumption equivalent increase,

[20]This figure are of the order of those reported in Levin *et al.* (2006) for a similar model but without nominal interest rate lower bound. The reason why they are much larger in these models is down to the welfare costs of price and (in the model of the latter paper) wage inflation, not included in the Lucas calculations, and to the estimated variances of the shocks.

a figure much larger than that found in most the current literature. Our finding endorses the conclusion reached by Adam and Billi (2006), discussed in the Introduction, namely that the lower bound constraint on the nominal interest rate increases the gains from commitment several fold.

7 Time Inconsistency and Policy Coordination in the Open Economy

We now turn to open economy aspects of the time-inconsistency problem. Following the seminal contribution of Obstfeld and Rogoff (1996), chapter 11, New Keynesian open economy DSGE modelling, the 'New Open Economy Macroeconomics', has been a highly active area.[21] Obstfeld and Rogoff developed a non-stochastic, perfect foresight two-country general equilibrium model with first flexible prices, and then price-rigidity. This model formed the basis for a wave of stochastic general equilibrium models that have been used to examine the potential gains from monetary policy coordination.[22].

Optimal policy can be formulated independently by each monetary authority. However In addition to the time-inconsistency problem there is a second classical problem first raised by Hamada (1976): in an open economy, rules designed for the single economy may perform badly in a world Nash equilibrium when all countries pursue similar optimal policies.

In the open economy the optimal monetary policy requires all policymakers to cooperate, maximizing an agreed global welfare, and to be able to commit not just with respect to each other but collectively with respect to the private sector too. These considerations lead to a number of possible equilibria depending on whether policymakers cooperate and can commit to the private sector and whether they can commit with respect to each other (i.e., can cooperate).

Consider symmetrical equilibria in the sense that all authorities can either commit or not with respect to the private sector. In the absence of any commitment mechanism for players all authorities must independently pursue discretionary policies (non-cooperation with discretion (ND)). If authorities can cooperate (i.e., can commit to each other) and can commit with respect to the private sector, then the socially optimal policy with respect to an agreed global objective function can be achieved (cooperation with commitment to the private sector, CC). The remaining possible equilibria are those where (for some

[21]See also Obstfeld and Rogoff (2000) and a recent survey by Lane (1999).

[22]See, for example, Benigno and Benigno (2001), Obstfeld and Rogoff (2002) Clarida *et al.* (2002), Pappa (2004), Liu and Pappa (2005), Batini *et al.* (2005)

reason) authorities can commit to each other but not to the private sector (cooperation with discretion, CD) or vice versa, they can commit to the private sector but not to each other (non-cooperation with commitment to the private sector, NC). Table 6 summarizes these four possibilities.

	Commitment	Discretion
Cooperation	CC	CD
Non-cooperation	NC	ND

Table 6: Possible Equilibria

These linear-quadratic dynamic game equilibria are formulated in Levine and Currie (1987a), Levine and Currie (1987b), Currie and Levine (1993), Currie *et al.* (1996)). *General* procedures, not specific to any one model, for their calculation and software for their computation have been developed (see Kemball-Cook *et al.* (1995).) In a two-bloc model the *potential gains from commitment in the absence of coordination* can be quantified by comparing the welfare in equilibria NC and ND. These 'gains' can be negative: as in Levine and Currie (1987b), for an ad hoc 'Old Keynesian' model *commitment without coordination may be counterproductive.* Similarly one can assess the *potential gains from coordination in the absence of commitment* by comparing equilibria CD and ND and revisit the possibility of *counterproductive cooperation* found by Rogoff (1985).

To realize the full potential gain from monetary policy coordination between the two blocs requires a combination of commitment and coordination; i.e., equilibrium CC and this can be be quantified by comparing CC with the non-cooperative alternatives, NC or ND. The first wave of the new Keynesian open economy models that revisited this old issue in the literature cited above suggested that these gains are not substantial compare with the gains from stabilization. Referring to table 6, Clarida *et al.* (2002) compare CD and ND and show there exists gains from CC if and only if $\sigma \neq 1$. Pappa (2004) and Benigno and Benigno (2001) compare CC and NC. Pappa (2004) shows gains are small and Benigno and Benigno (2001) show that CC can be sustained as an NC equilibrium by delegation to a central bank with an appropriate loss function. Finally Currie and Levine (1993) compare CC and ND, but using an ad hoc model and utility function.

These conclusions are based on either the earlier generation of ad hoc models and loss functions, or on very simple micro-founded models. In the words of Canzoneri *et al.* (2005), "What is yet to come" is the reassessment of these gains using empirical DSGE models incorporating various persistence mechanisms, incomplete exchange rate pass-through, incomplete financial markets, 'home bias', a non-traded sector and wage stickiness, all factors that could well affect both commitment and coordination gains. In particular:

1. **Persistence Mechanisms**

   In order to obtain a better fit with data output persistence can be incorporated by adding habit in consumption and/or labour supply and indexing into Calvo contracts (see Batini *et al.* (2005)).

2. **The Exchange-Rate Pass-Through Mechanism and Incomplete Markets**

   Devereux and Engel (2002) in their solution to the 'exchange rate disconnect' puzzle propose three key elements of the solution: the possibility of local currency pricing (LCP); heterogeneity in the way exported goods are priced i.e. some involve LCP, whereas others involve producer currency pricing (PCP); thirdly, incomplete markets and 'noise traders' whose expectations are conditionally biased.

3. **Home Bias**

   Households typically may have a preference for goods produced in the home country. However Corsetti *et al.* (2002) shows that this is insufficient in its own right to explain why the correlation between relative consumption and the real exchange rate is negative for many countries.

4. **Traded and Non-traded Sectors**

   This feature which introduces the Balassa-Samuelson effect is stressed in Corsetti *et al.* (2002), Natalucci and Ravenna (2002), Liu and Pappa (2005) and Canzoneri *et al.* (2005). The former construct a model with a non-traded sector and incomplete asset markets. Departures from PPP occur through the existence of a distribution sector (as in Monacelli (2003)) but *there is no price-stickiness in their model* . Despite this limitation their model with different productivity processes in the traded and non-traded sectors accounts for both the exchange rate disconnect puzzle and a low degree of risk-sharing with a negative correlation between relative consumption and the real exchange rate. These features are combined with a significant (and negative) transmission of a productivity increase in one country.

5. **Wage Stickiness**

   As Erceg *et al.* (2000) and Blanchard and Gali (2005) argue, wage plus price stickiness are necessary to avoid the implausible 'divine coincidence' property that stabilizing inflation also stabilizes the output gap. Note however that divine coincidence is also removed by other means, such as the ad hoc mark-up shocks that are typically added to the Phillips curves at the Bayesian estimation stage and by the non-separability of money and consumption.

8 Conclusions

Macroeconomics research has changed profoundly since the Kydland-Prescott seminal paper. In order to address the Lucas Crtique, modelling now is based on micro-foundations treating agents as rational utility optimizers.[23] Bayesian estimation has produced models which are more data consistent than those based simply on calibration. With micro-foundations and new linear-quadratic techniques, normative policy based on welfare analysis is now possible. In the open economy, policy involves a 'game' with policymakers and private institutions or private individuals as players. This paper has attempted to reassess the Kydland-Prescott contribution in the light of these developments. Despite this sea-change the relevance of the time-inconsistency problem remains. Indeed, since time-inconsistency rests on the existence of forward-looking, rational agents, the use of micro-foundations which introduces more forward-looking behaviour, has increased its relevance.

The gains from commitment and how to sustain them will continue to preoccupy economists in all areas of the subject. For macroeconomists, perhaps the most fruitful area for future research will be in the open-economy aspects where two commitment problems arise: that between authorities such as central banks and that between these institutions and the private sectors. What is yet to come, then, is a study of of these issues in the context of the 'New Open Economy Macroeconomics.'

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[23]Though Calvo-contracts, especially with indexing, represents something of a compromise, that reconciles rigor with data consistency.

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A Linearization about the Zero-Inflation Steady State

We linearize about the deterministic zero-inflation steady state. Output is then at its sticky-price, imperfectly competitive natural rate and from (25) the nominal rate of interest is given by $\bar{\imath} = \frac{1}{\beta} - 1$. Define all lower case variables as proportional deviations from this baseline steady state except for rates of change which are absolute deviations.[24] Then the linearization takes the form:

$$\pi_t = \frac{\beta}{1+\beta\gamma}E_t\pi_{t+1} + \frac{\gamma}{1+\beta\gamma}\pi_{t-1} + \frac{(1-\beta\xi)(1-\xi)}{(1+\beta\gamma)\xi}mc_t \tag{A.1}$$

$$mc_t = \frac{\sigma}{(1-h_C)}(c_t - h_C c_{t-1}) + \frac{\phi}{(1-h_L)}(l_t - h_L l_{t-1}) - a_t \tag{A.2}$$

$$l_t = y_t - a_t \tag{A.3}$$

$$c_t = \frac{h_C}{1+h_C}c_{t-1} + \frac{1}{1+h_C}E_t c_{t+1} - \frac{1-h_C}{(1+h_C)\sigma}(i_t - E_t\pi_{t+1}) \tag{A.4}$$

$$y_t = c_y c_t + (1-c_y)g_t \quad \text{where } c_y = \frac{C}{Y} \tag{A.5}$$

$$g_{t+1} = \rho_g g_t + \epsilon_{g,t+1} \tag{A.6}$$

$$a_{t+1} = \rho_a a_t + \epsilon_{a,t+1} \tag{A.7}$$

Variables y_t, c_t, mc_t, a_t, g_t are proportional deviations about the steady state. $[\epsilon_{g,t}, \epsilon_{a,t}]$ are i.i.d. disturbances. π_t and i_t are absolute deviations about the steady state. For later use we require the *output gap* the difference between output for the sticky price model obtained above and output when prices are flexible, $\hat{y}_t$ say. The latter, obtained by setting $\xi = 0$ in (A.1) to (A.5), is in deviation form given by[25]

$$\hat{mc}_t = 0 = \frac{\sigma}{(1-h_C)}(\hat{c}_t - h_C\hat{c}_{t-1}) + \frac{\phi}{(1-h_L)}(\hat{l}_t - h_L\hat{l}_{t-1}) - a_t \tag{A.8}$$

$$\hat{l}_t = \hat{y}_t - a_t \tag{A.9}$$

$$\hat{y}_t = c_y\hat{c}_t + (1-c_y)g_t \tag{A.10}$$

We can write this system in state space form as

$$\begin{bmatrix} \mathsf{z}_{t+1} \\ E_t\mathsf{x}_{t+1} \end{bmatrix} = A\begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} + Bi_t + C\begin{bmatrix} \epsilon_{g,t+1} \\ \epsilon_{a,t+1} \end{bmatrix} \tag{A.11}$$

$$\begin{bmatrix} y_t \\ \hat{y}_t \end{bmatrix} = E\begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} \tag{A.12}$$

where $\mathsf{z}_t = [a_t, g_t, l_{t-1}, \hat{l}_{t-1}, c_{t-1}, \hat{c}_{t-1}, \pi_{t-1}]$ is a vector of predetermined variables at time t and $\mathsf{x}_t = [c_t, \pi_t]$ are non-predetermined variables. Rational expectations are formed

[24] That is, for a typical variable X_t, $x_t = \frac{X_t - \bar{X}}{\bar{X}} \simeq \log\left(\frac{X_t}{\bar{X}}\right)$ where $\bar{X}$ is the baseline steady state. For variables expressing a rate of change over time such as i_t, $x_t = X_t - X$.

[25] Note that the zero-inflation steady states of the sticky and flexi-price steady states are the same.

π_t	producer price inflation over interval $[t-1, t]$
i_t	nominal interest rate over interval $[t, t+1]$
mc_t	marginal cost
l_t, $\hat{l}_t$	consumption with sticky prices and flexi-prices
y_t, $\hat{y}_t$	output with sticky prices and flexi-prices
l_t, $\hat{l}_t$	employment with sticky prices and flexi-prices
$x_t = y_t - \hat{y}_t$	output gap
$a_{t+1} = \rho_a a_t + \epsilon_{a,t+1}$	AR(1) process for factor productivity shock, a_t
$g_{t+1} = \rho_g g_t + \epsilon_{g,t+1}$	AR(1) process government spending shock, g_t
β	discount parameter
γ	indexation parameter
h_C, h_N	habit parameters
$1 - \xi$	probability of a price re-optimization
σ	risk-aversion parameter
ϕ	disutility of labour supply parameter

Table 1: Summary of Notation (Variables in Deviation Form)

assuming an information set $\{z_s, x_s\}$, $s \leq t$, the model and the monetary rule. Table 1 provides a summary of our notation.

Model			Model 1			Model 2			Model 3			Model 4			Model 5			Model 6			Model 7			Model 8		
Para/ Prior mean / Distribution			Post. mean	Post. med.	Conf. inter. 90%	Post. mean	Post. med.	Conf. inter. 90%	Post. mean	Post. med.	Conf. inter. 90%	Post. mean	Post. med.	Conf. inter. 90%	Post. mean	Post. med.	Conf. inter. 90%	Post. mean	Post. med.	Conf. inter. 90%	Post. mean	Post. med.	Conf. Inter. 90%	Post. mean	Post. med.	Conf. inter. 90%
γ	0.7	beta										0.52	0.51	(0.35 0.68)				0.65	0.65	(0.48 0.82)	0.53	0.53	(0.37 0.68)	0.63	0.63	(0.46 0.81)
λ	0.2	gam	0.15	0.15	(0.07 0.24)	0.22	0.20	(0.07 0.35)	0.12	0.12	(0.06 0.19)	0.18	0.17	(0.07 0.26)	0.16	0.14	(0.04 0.26)	0.53	0.52	(0.24 0.81)	0.13	0.15	(0.03 0.20)	0.40	0.38	(0.18 0.61)
h_C	0.7	beta				0.85	0.86	(0.75 0.96)							0.85	0.86	(0.77 0.96)	0.84	0.85	(0.72 0.95)				0.84	0.85	(0.73 0.95)
h_N	0.7	beta							0.65	0.65	(0.49 0.80)				0.66	0.67	(0.51 0.82)				0.66	0.67	(0.51 0.82)	0.59	0.60	(0.44 0.74)
σ	1.5	gam	0.06	0.06	(0.02 0.11)	3.48	3.47	(1.70 5.13)	0.10	0.10	(0.03 0.15)	0.07	0.06	(0.02 0.13)	3.46	3.38	(1.70 5.13)	3.52	3.40	(0.79 5.22)	0.12	0.13	(0.04 0.18)	3.49	3.35	(1.83 5.26)
ρ_g	0.7	beta	0.94	0.94	(0.90 0.98)	0.95	0.95	(0.92 0.98)	0.94	0.94	(0.90 0.98)	0.94	0.94	(0.90 0.98)	0.94	0.95	(0.91 0.98)	0.95	0.95	(0.92 0.98)	0.94	0.95	(0.91 0.98)	0.95	0.98	(0.91 0.98)
ρ_a	0.7	beta	0.59	0.59	(0.44 0.72)	0.87	0.87	(0.81 0.93)	0.52	0.53	(0.37 0.67)	0.30	0.30	(0.14 0.48)	0.86	0.87	(0.80 0.93)	0.89	0.89	(0.83 0.93)	0.25	0.24	(0.10 0.37)	0.88	0.88	(0.83 0.93)
ρ_i	0.8	beta	0.88	0.88	(0.83 0.93)	0.76	0.76	(0.68 0.84)	0.85	0.85	(0.79 0.91)	0.88	0.88	(0.83 0.93)	0.78	0.78	(0.71 0.85)	0.72	0.72	(0.63 0.83)	0.85	0.86	(0.78 0.91)	0.75	0.76	(0.67 0.84)
θ	1.7	gam	1.74	1.69	(1.00 2.35)	2.34	2.30	(1.54 3.12)	2.07	2.01	(1.36 2.85)	1.88	1.81	(1.06 2.62)	2.17	2.15	(1.31 2.94)	2.44	2.42	(1.63 3.19)	2.00	1.97	(1.27 2.65)	2.42	2.36	(1.62 3.15)
r^*	2	gam	1.77	1.83	(0.88 2.53)	2.00	2.01	(1.35 2.64)	1.83	1.85	(1.06 2.60)	1.82	1.83	(0.97 2.62)	1.99	1.99	(1.38 2.69)	1.98	1.98	(1.32 2.64)	1.85	1.82	(1.08 2.67)	1.94	1.96	(1.27 2.60)
π^*	4	gam	3.01	3.02	(2.42 3.57)	2.81	2.82	(2.09 3.50)	2.94	2.95	(2.36 3.44)	3.07	3.04	(2.38 3.79)	2.83	2.84	(2.08 3.56)	2.73	2.72	(1.95 3.57)	3.03	3.00	(2.37 3.66)	2.80	2.82	(2.00 3.59)
SD of shocks																										
ε_g	1.7	invg	2.58	2.56	(2.23 2.93)	2.64	2.64	(2.26 3.01)	3.04	3.00	(2.42 3.61)	2.68	2.67	(2.29 3.06)	2.65	2.64	(2.26 3.00)	2.67	2.65	(2.29 3.04)	3.19	3.19	(2.52 3.72)	2.67	2.64	(2,30 3.04)
ε_a	1.7	invg	0.94	0.87	(0.46 1.47)	1.67	1.58	(1.02 2.29)	1.05	0.95	(0.53 1.52)	1.05	0.94	(0.49 1.62)	1.44	1.35	(0.83 2.21)	1.23	1.19	(0.81 1.65)	1.40	0.88	(0.57 3.39)	1.07	1.03	(0.70 1.41)
ε_e	1	invg	0.15	0.14	(0.13 0.17)	0.16	0.16	(0.14 0.18)	0.15	0.15	(0.13 0.17)	0.15	0.15	(0.13 0.18)	0.16	0.16	(0.14 0.19)	0.18	0.18	(0.15 0.21)	0.15	0.15	(0.13 0.17)	0.18	0.18	(0.15 0.21)
Median of ζ (contract length)			0.68(3.14)			0.64(2.80)			0.71(3.45)			0.61(2.54)			0.69(3.23)			0.41(1.69)			0.62(2.66)			0.46(1.87)		
Log marginal density			-262.992			-260.582			-263.957			-274.980			-260.844			-260.764			-269.707			-262.658		
Prob.			3.15%			35.05%			1.2%			0			26.97%			29.22%			0			4.4%		

Table 2: Priors, Posterior Estimates and Log Marginal Densities